Workshop 2024/04/13

Questions only

Problem 1:

a. Find all pairs (N, M) where N and M are primes and $5N + 11M = 102$

Suppose that the pair (N, M), $N > 0, M > 0$ satisfies $5N + 11M = 222$.

b. What is the largest possible value of $N + M$?

c. What is the smallest possible value of $N + M$?

Problem 2:

a) How many factors are there of the following two numbers: 48, 360?

b) What is the sum of all the factors of these numbers: 48, 360?

Problem 3: convert $0.41666\cdots$ to a fraction.

Problem 4:

Consider a positive number N.

a. Suppose that N is divisible by 9 with no remainder, what is the minimum number of digits of N if its digit sum is greater than 90 but less than 100?

b. Suppose that N is divisible by 5 with no remainder. What is the minimum number of digits of N if its digit sum is 99?

c. Suppose that N is divisible by 4 with no remainder. What is the minimum number of digits of N if its digit sum is 97?

d. Suppose that N is divisible by 3 with no remainder. What is the minimum number of digits of N if its digit sum is 96?

e. Suppose that N is divisible by 2 with no remainder. What is the minimum number of digits of N if its digit sum is 95?

Problem 5:

Suppose that light travels at speed of $300{,}000\frac{km}{sec}$.

a. How many kilometres will it travel in one minute.

b. If the Earth is $150{,}000{,}000\ km$ away from the sun, how many minutes it takes for the sun light to reach Earth (round the answer to the nearest minute).

c. If Pluto is 6 billion km away from the sun, how many hours it takes the light to travel from the Sun to Pluto.

d. Suppose that one of the nearest stars is located $60{,}000{,}000{,}000{,}000\ km$ away. Approximately, how many years it takes for the Sun light to reach that star?

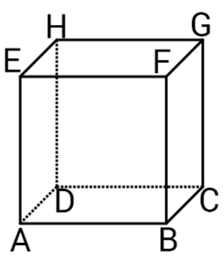
Problem 6:

a. What is the value of the 10-th term of the following arithmetic sequence: $\{-7, -3, 1, \cdots\}$?

a. What is the value of the 6-th term of the following geometric sequence: $\{-1, 2, -4, \cdots\}$?

Problem 7:
Suppose that 5 dice are rolled.
a. How many different sums can be rolled?
b. How many of these sums are odd?
c. How many of these sums are even?
d. How many of these sums are multiples of 5?
e. How many of these sums are prime numbers?
f. What is the probability that the sum is 7?
Suppose that 2 fair dice are rolled.
g. if you know that at least one of the dice shows an odd number, what is the probability that the sum is odd?
h. if you know that at least one of the dice shows an odd number, what is the probability that the sum is even?
i. if you know that at least one of the dice shows an even number, what is the probability that the sum is odd?
j. if you know that at least one of the dice shows an even number, what is the probability that the sum is even?

Problem 8: Box with sides x, y, z.
A, C, E, G are the corners of the box.
The lines AB, CG, DH are edges.
$ABFE$ is one of the faces of the box.
Suppose that the values of the edges of the box below are $x = 2,\ y = 3$, and $z = 4$.
Find the following:
a. What is the volume of the box?
b. How many faces does the box have?
c. How many edges does the box have?
d. How many corners does the box have?
e. What are the areas of each of its face? And, what is the total area of all its faces?
f. What is the sum of all its edges?

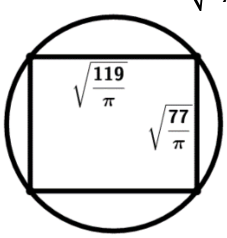


Problem 9: On the ground there is a big container of water that is initially empty. The container has can be filled with 3 taps, (that can operate at the same time if desired), named A, B, and C.
Tap A operating alone can fill it up in 6 hours, tap B operating alone can fill it up in 8 hours, and tap C operating alone can fill it up in 24 hours. How many hours will it take to fill the container up if all 3 taps are turned on?

Problem 10:
a. What is the sum of all angles (in degrees) of a regular polygon with 11 sides?
b. What is the size (in degrees) of each of the angles of a regular polygon with 12 sides?
c. What is the largest number of sides of a regular polygon if all of it angles are whole numbers (in degrees)?
b. If the value, (in degrees), of each of the angles of the regular polygon is a prime number, what is minimum number of sides that such a polygon can have?

Problem 11:
a. How many positive numbers N satisfy $N^4 < 1{,}000{,}000$?

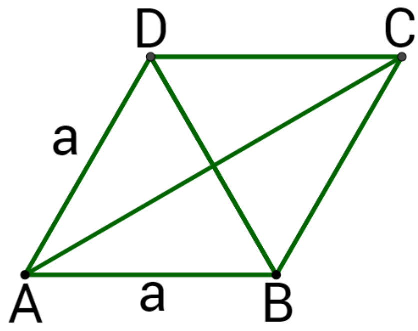
Problem 12:
A rectangle with sides $\sqrt{\frac{119}{\pi}}$ and $\sqrt{\frac{77}{\pi}}$ is inscribed in a circle. What is the area of the circle?



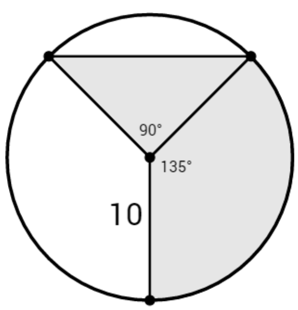
Problem 13:
For every 1400 boys members of the Canadian Arts and Science Club there are 2023 girls that are members of the club. What percentage of the club membership are the girls? Round your answer to the nearest whole number.

Problem 14:
Two teams, A and B, compete in a basketball championship. The probability of Team A to win a game is 80%, and the probability of Team B to win a game is 20%, (no ties). The first team to win 3 games in total wins the championship. What is the probability that it will take only 3 games to decide the championship? Express the answer as a fraction in lowest terms.

Problem 15:
Below is a rhombus with sides $a = 7$. Its short diagonal satisfies $BD = a = 7$. What is the square value of the long diagonal, (i.e. the value of AC^2) ?



Problem 16:
The shaded section of the circle of radius 10 consists of a right triangle and a sector of $135°$. Find the area of the shaded section. Use $\pi = 3.14$, and round your answer to the nearest whole number.



Problem 17:
$\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x} = 3$. Express x as a fraction in lowest terms.

Problem 18:

Jill drives a fuel-efficient car that consumes, on average, 6 litres of fuel per hour. When she started driving, the fuel tank was full. After driving T hours she stopped and added 10 litres of fuel so that the tank was 85% full. Then, she drove $\frac{T}{2}$ hours, stopped again, and filled the tank with 17 more litres of fuel so that the fuel tank was full again. How many liters of fuel can a full tank hold? Round your answer to the nearest whole number.

Problem 19:

The parliament proposes an increase of 400% to the current carbon tax to a new tax rate of $120 per tonne. What is the current tax rate per tonne (in $)?

Problem 20:

In how many ways can you pay 80 cents using any combination of 5, 10, and 25 cent coins?

Problem 21:

N is the smallest positive whole number such that all the following conditions are satisfied: $\{a, b, c, d, e, f\}$ is a set of 6 different primes, $N = a + b + c = d + e + f$, $a < b < c$, and $d < e < f$. What is the maximum possible value of $c - a$?

Problem 22:

Dan read a 650 page book in the following way. On the first day he read every second page of the book starting at page 1 (i.e. he read pages 1, 3, 5, and so on). On the second day he read every third page of the book starting at page 1 (i.e. he read pages 1, 4, 7, and so on). How many of the pages did he read twice?

Problem 23:

What is the sum of all factors of 2023? (**Hint**: 2023 is not a prime number).

Problem 24

A triangle has sides L, M, and N, where $0 < L < M < N < 12$ are all whole numbers. The perimeter of the triangle is P. How many different values of P are there?

Problem 25:

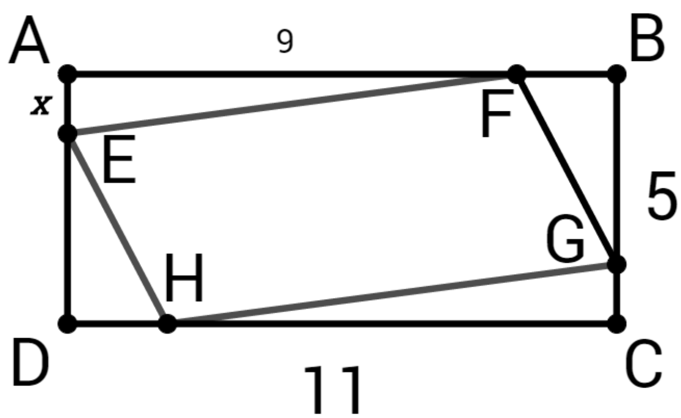
$N + (N+1) + (N+2) + \cdots + 2023 = 94 \times 1000$, and N>0. What is the Value of N?

Problem 26:

Frank wrote down the sum of the digits of every number from 1 to 1000. (Examples: for the number 2 he wrote 2, for the number 24 he wrote 6 because $2+4=6$, for the number 200 he wrote 2 because $2+0+0=2$, and for the number 550 he wrote 10 because $5+5+0=10$). How many times did he write the digit 0?

Problem 27:

$ABCD$ is a rectangle with sides 11 and 5. $EFGH$ is a parallelogram. $AE = x, AF = 9$. The area of $EFGH$ is $\frac{2}{3}$ of the area of $ABCD$. What is the area of ΔDEH? Express the answer as a fraction in lowest terms.



Problem 28:

There is a pile of 7 cards numbered $1,2,3,\cdots,7$ on the table. Gloria takes 3 different cards at random from the pile and writes down the sum of these 3 cards. What is the probability that the sum is a multiple of 3? Express the answer as a fraction in lowest terms.

Problem 29:

Eric takes 4 times longer to paint a ceiling than to paint a wall. He charges 20% more per hour to paint a ceiling than to paint a wall. Eric painted 5 ceilings and 12 walls. His hourly charge per wall was $40 and his total earning was $2520. How many hours did he work in total?