ELMACON Preparation Session

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1 Problem Set A

1. A man has a rectangular patio in his garden. He decides to enlarge it by increasing both the length and width by 10%. What is the percentage increase in its total area?

   1. _____

2. Joe has several identical balls and several identical dice. Together 4 balls and 3 dice weigh 37 grams, while 3 balls and 4 dice weigh 33 grams. What is the combined weight, in grams, of one ball and one dice.

   2. _____

3. Let $x = N + 2 \times N + 3 \times N + \cdots + 100 \times N$. What is the smallest integer N for which x is a perfect square?

   3. _____

4. To make lawn fertilizer, a manufacturer mixes nitrogen, phosphoric acid, and potash in the ratio of 3 : 8 : 17. If a batch of the mixture contains 6kg of nitrogen, how much potash does it contain?

   4. _____

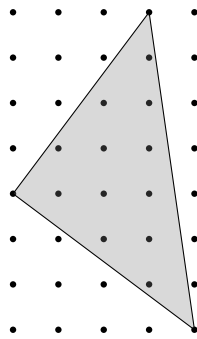5. If the number pattern shown below is continued, find the third number in row 10.

$$
\begin{array}{lccccccccc}
n = 0: & & & & & 1 & & & & \\
n = 1: & & & & 1 & & 1 & & & \\
n = 2: & & & 1 & & 2 & & 1 & & \\
n = 3: & & 1 & & 3 & & 3 & & 1 & \\
n = 4: & 1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

$$\vdots$$

5. _____

6. Jane was born on June 30, 1994.Alex was born on June 3, 1995. Find the number of days between their birth dates (not including their birth dates).

6. _____

7. In the product $P8 \times 3Q = 2730$, the letters P and Q represent different digits from 1 to 9. Find $P + Q$.

7. _____

8. The digits of 4795 can be rearranged to form different numbers. What is the sum of the largest and smallest numbers that may be formed?

8. _____

9. The number 1 is both the square of an integer and the cube of an integer. What is the next larger number which is both a square and cube of a positive integer?

9. _____

10. Alf rolls two dice and adds the numbers that come up. What is the probability that the sum is 9?

10. _____

11. A dog and a rabbit are 160 meters apart. The dog chases the rabbit. For every 9 meters that the dog runs, the rabbit runs 7 meters. Find the distance, in meters, that the dog must run to catch the rabbit.

11. _____

12. Find the whole number N between 0 and 20 for which the following steps will give the output $A = 9$.

2

1. Use the starting number N to build a new number M:

   - If N is 9 or smaller, let M be $N + 10$
   - If N is 10 or greater, let M be $N - 5$

2. Divide M by 16, and call the remainder R.

3. Multiply R by 3. Call the result your output, A.

12. _____

13. Find the area of the shaded triangle in the sketch below. (The distance from each point to its nearest neighbors is one unit).

13. _____

14. Eve, Jane and Amy have different collections and different clothing.

   1. One of them collects cards
   2. One of them wears a red shirt
   3. Amy collects stamps
   4. The one wearing a white shirt collects rocks
   5. Eve is not wearing a blue shirt and does not collect rocks

   What color is Eve's shirt?

14. _____